



University of  
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Science

# ***Lecture 6: Informed Search, Part 2***

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# Today on CS310

- Searching graphs with costs attached to the edges
- Uniform cost (cheapest first) search
- Reminder of all the different search methods
- Why  $A^*$  returns the optimal path
- How to design a good heuristic function

# Uniform Cost Search

- If there are costs attached to the edges and we want the lowest cost plan, we use *uniform cost search*
- For each path on the agenda, we store the cost of the path.
- When selecting a path to expand, select the one with the lowest cost
- Returns the plan with the lowest cost
- Note: this is not a heuristic search method!

# Heuristic Search

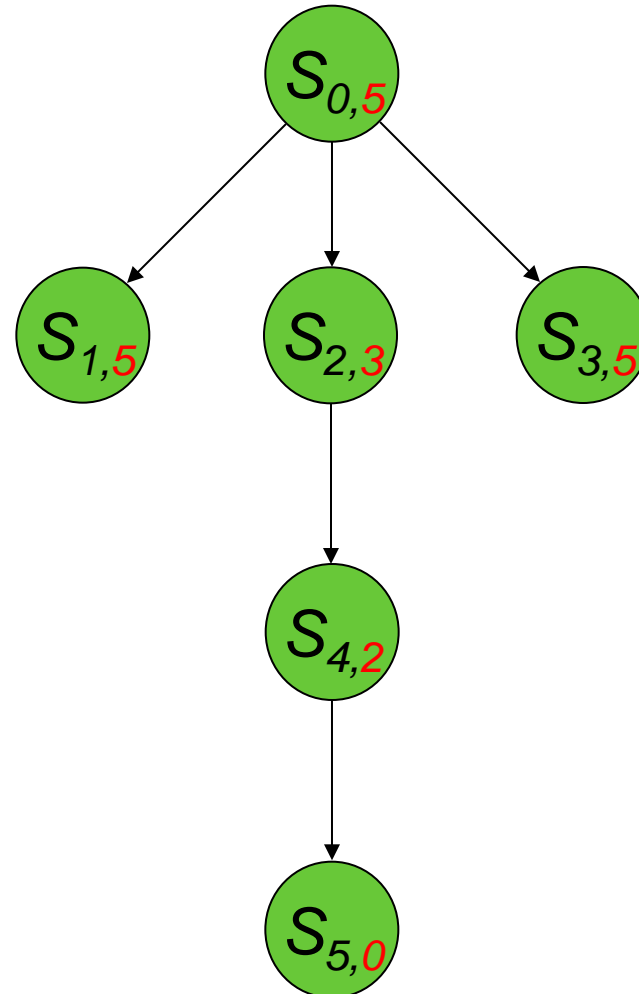
- We can often estimate the distance from  $S_i$  to  $G$  by using a **heuristic function**,  $h(S_i, G)$
- The function **efficiently** compares the two states and tries to get an estimate of how many moves remain without doing any searching
- For example, in the blocks world, all blocks that are stacked up in the correct place never have to move again; all blocks that need to move that are on the table only need to move once; and all other blocks only need to move at most twice:

$$h(S_i, G) = 2 * B_{bad} + 1 * B_{table} + 0 * B_{good}$$

# Enforced Hill Climbing

- The easiest way to use a heuristic estimate to search is to require that every single move we make takes us strictly closer to the goal
- The form of search doesn't even require an agenda, since at each decision point, we take the action that looks best to us and repeat until we're done
- Problems: dead ends, plateaus, solution quality (i.e. the number of steps can be very poor)
- Used to good effect in the FF planner (which reverts to best-first search if enforced hill climbing fails)

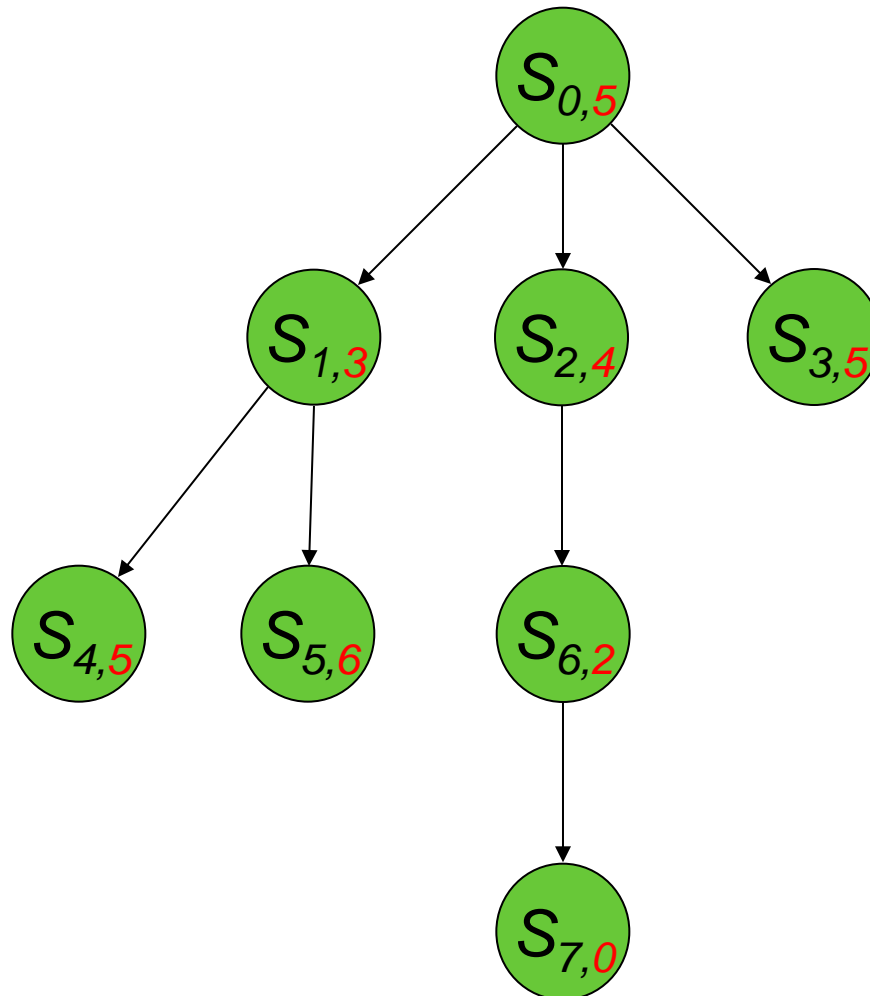
# Enforced Hill Climbing



# Best-First Search

- Enforced hill climbing is great when it works, but for some problems it's better to keep track of the nodes we haven't yet expanded, using the agenda
- We can then use the heuristic function to determine which node to expand next
- As new states are discovered, we add them to the agenda and record the value of the heuristic function
- When we pick the next node to explore, we choose the one which has the *lowest value* for the heuristic function (i.e. the one that looks nearest to the goal)

# Best-First Search





# Best-First Search

- To get best-first search, pick the **best** node on the agenda as the one to be explored next:

let Agenda = [ $S_0$ ]

while Agenda  $\neq$  [] do

    let Current = **Best**(Agenda)

    let Agenda = Rest(Agenda)

    if Goal(Current) then return (“Found it!”)

    let Next = NextStates(Current)

    let Agenda = Agenda + Next

# Best-First Search and Algorithm A

- Best-first search can speed up the search by a very large factor, but can it isn't guaranteed to return the shortest solution
- When deciding to expand a node, we need to take account of how long the path is so far, and add that on to the heuristic value:

$$f(S_i, G) = g(S_0, S_i) + h(S_i, G)$$

- This will give a search which has elements of both breadth-first search and best-first search
- This type of search is called “Algorithm A”

# Algorithm A\*

- If  $h(S_i, G)$  never over-estimates the distance from  $S_i$  to the goal, it is called an **admissible** heuristic
- If  $h(S_i, G)$  is admissible, then Algorithm A will **always** return the shortest path (like breadth-first search) but will omit much of the work if the heuristic function is informative
- The use of an admissible heuristic turns Algorithm A into **Algorithm A\***
- Uses: problem solving, route finding, path planning in robotics, computer games, etc.

## Why is A\* Optimal?

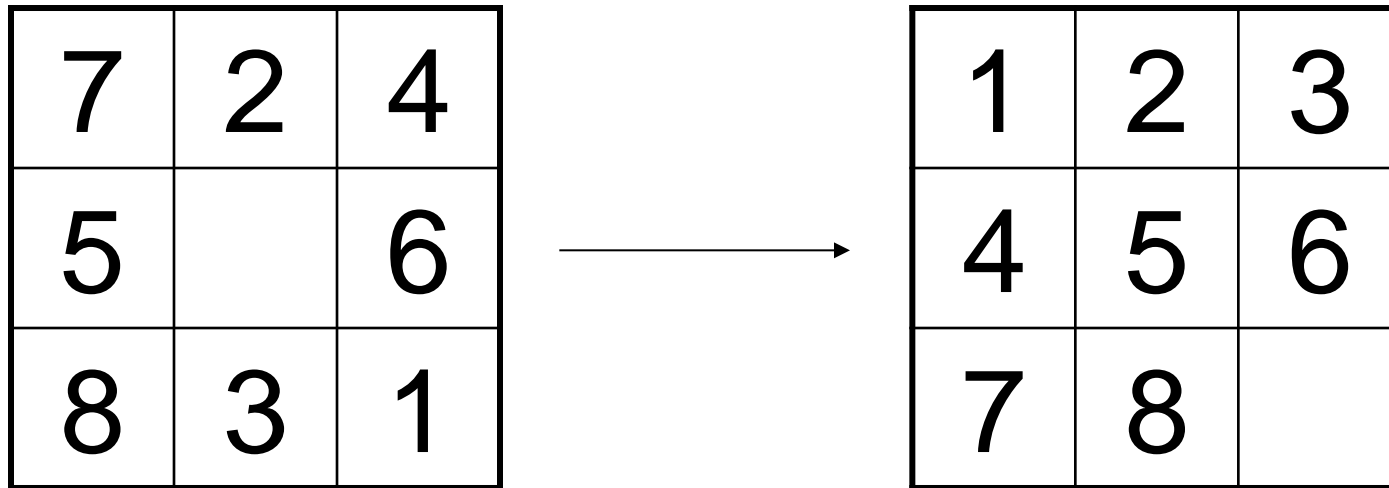
- Suppose a suboptimal goal node,  $S_k$ , appears in the agenda – we haven't selected it yet, so we don't yet know that it's a goal node
- Also on the agenda, there must be a node,  $S_i$  which is on the optimal path from  $S_0$  to the goal state
- Since the heuristic function,  $h(S_i, G)$ , is admissible, this means:

$$g(S_0, S_k) + h(S_k, G) > g(S_0, S_i) + h(S_i, G)$$

so  $S_k$  will never be selected over  $S_i$  for expansion.

# Heuristic Functions

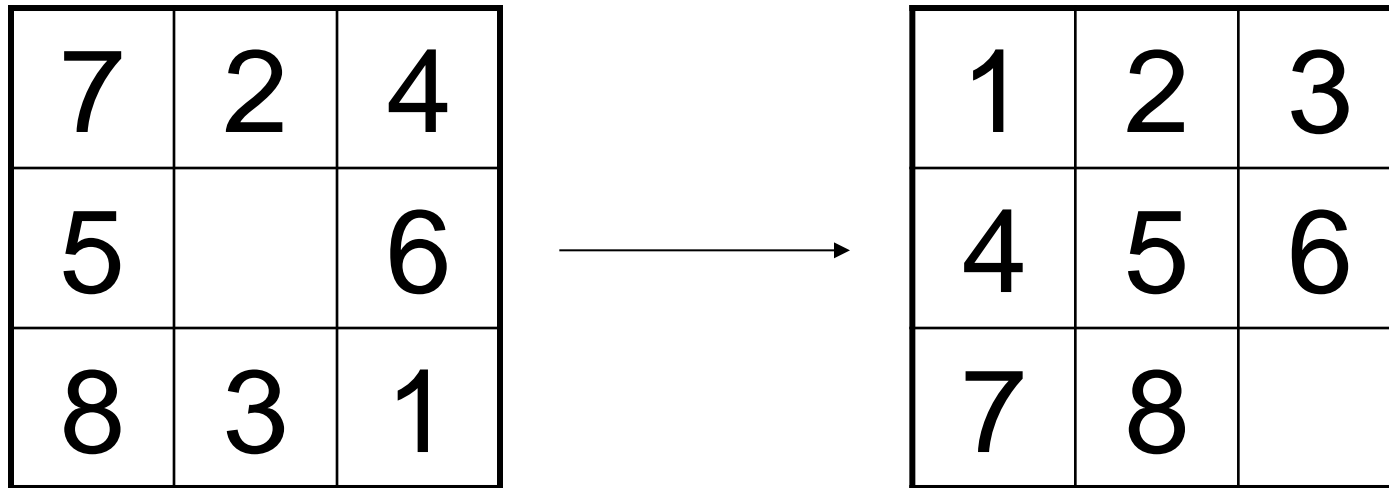
- Consider the 8-puzzle:



- Can we come up with a good admissible heuristic function for this problem?

# Heuristic Functions

- Consider the 8-puzzle:



- $h_1$  = number of misplaced tiles  
Exercise: calculate this for the above problem

# Problem Relaxation

- We can do better than this!
- How about if we somehow *relax* the problem and then solve the relaxed problem *exactly*?
- The relaxed problem needs to be tractable, i.e. can be solved in polynomial time
- Relaxing the problem means allowing the laws of physics to change, removing constraints from the problem, generally making it easier to find a solution
- How could we relax the 8-puzzle?

# Summary of Search

- Uninformed search methods: depth-first, breadth-first, iterative deepening, uniform cost search.
- Informed search methods, using a heuristic: enforced hill-climbing, best-first, algorithm A, algorithm A\*.
- Heuristic search is generally faster, but heuristics are problem dependent
- Algorithm A\* requires an admissible heuristic to give an optimal solution