Lecture 7: Adversarial Search

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Learning Outcomes for Today

After today's lecture, you will know:

- How two-player zero-sum games are an extension of the single agent problems we've examined so far
- What a game tree looks like
- How we can solve a game tree by assuming that the opponent plays rationally
- How the Minimax algorithm works

Which Problems can we Solve?

The task environments which are suitable for the search algorithms we've looked at so far are:

- Fully observable (the easy option)
- Deterministic (the easy option)
- Sequential (the normal option)
- Static (the easy option)
- Discrete (the easy option)
- Single agent (the easy option)

Games

- To play a game, we need to relax the assumption that only one agent can change the state of the world.
- Game theory: any environment with multiple agents in it can be regarded as a game.
- In AI, games are usually what game theorists would call deterministic, turn-taking, two-player, zero-sum games of perfect information.
- Examples: chess, checkers, Connect 4, shogi, Othello, go, tic-tac-toe, dots and boxes, …

Features of these Games

- Fully observable: game state is visible to both players
- Deterministic: no element of chance
- Sequential: action taken now affects future choices
- Static: the world doesn't change during deliberation
- Discrete: the game state can be represented exactly using a finite representation
- Multi agent: in the search, we have to allow for the fact that our opponent can also affect the game state when it is their turn, and will be planning against us

Representing a Game

We need:

- An initial state, including who plays first
- A successors (state) function, like next-states (state)
- A terminal test which determines whether a given state is a end state of the game (i.e. the game is over)
- A utility function – for terminal states only, this is the reward each player gets (e.g. +1 for win, −1 for lose)
- In a zero-sum game, the utilities of the end states sum to the same amount for the two players

The Two Players: Max and Min

- Let's call the two players Max and Min: Max goes first
- Max's task is to maximise Max's reward
- Min's task is to minimise Max's reward (and therefore maximise Min's reward)
- Let's say Max can take actions *a*, *b* or *c* – which one will give Max the best reward when the game ends?
- To answer that question, we need to explore the game tree to a *sufficient depth*, and *assume that Min plays optimally* to minimise the reward that Max gets

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An Example

- Four coins in a row, each player can pick up one coin or two coins.
- The player who picks up the last coin wins.
- Max plays first. What move should Max make?

The Minimax Value

- The minimax value of a node is the utility of the node if the node is a terminal
- If the node is a non-terminal Max node, the minimax value of the node is the maximum of the minimax values of all of the node's successors
- If the node is a non-terminal Min node, the minimax value of the node is the minimum of the minimax values of all of the node's successors
- Recursive definition: results in a depth-first traversal of the game tree

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The Minimax Algorithm

```
Max-Value(state) returns a utility value
    if Terminal-Test(state) then return Utility(state)
    v ← MinimalGameValue
    for s in Successors(state) do
        v ← Max(v, MinValue(s))
    return v

Min-Value(state) returns a utility value
    if Terminal-Test(state) then return Utility(state)
    v ← MaximalGameValue
    for s in Successors(state) do
        v ← Min(v, Max-Value(s))
    return v
```

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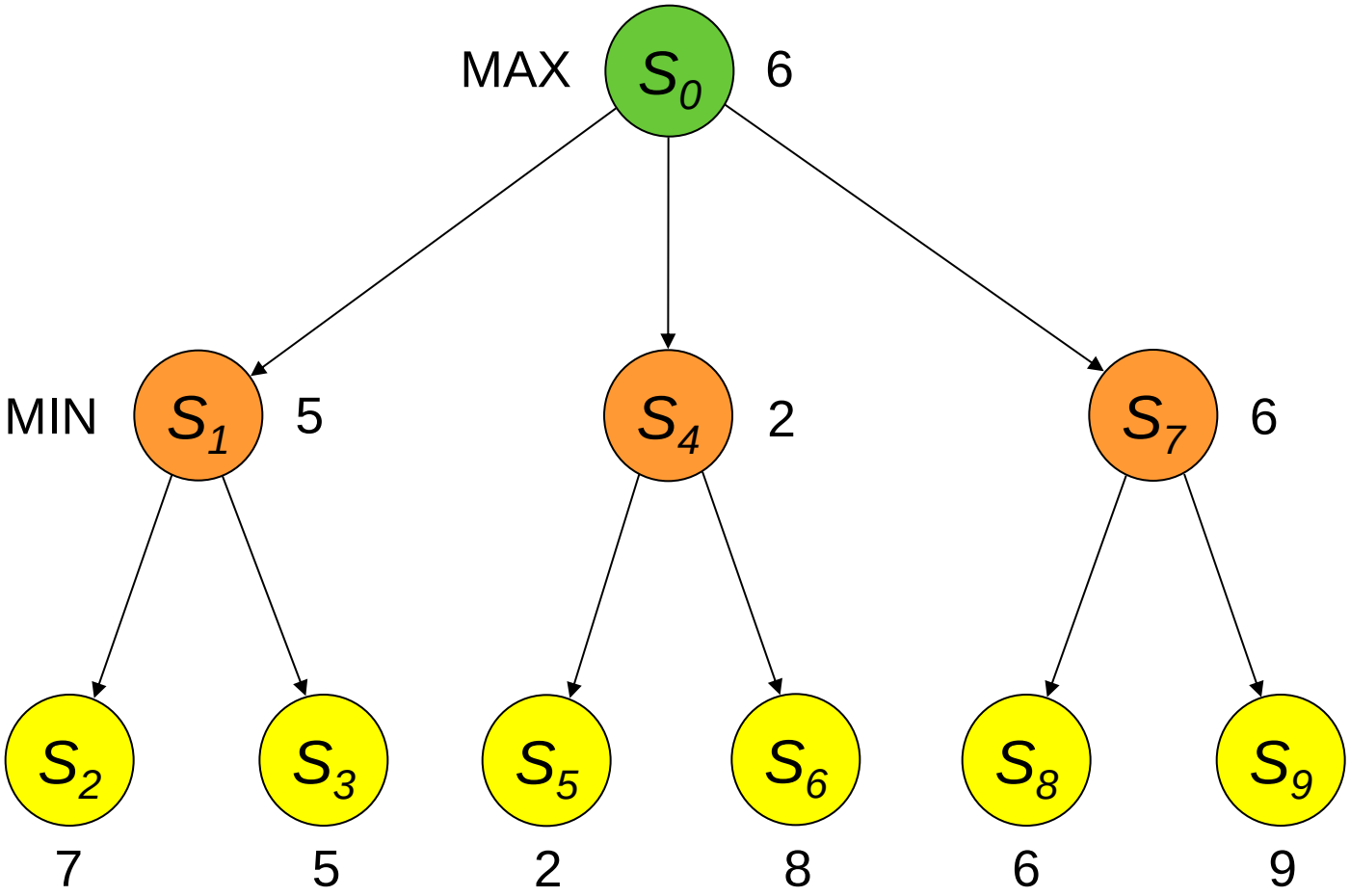
The Minimax Algorithm



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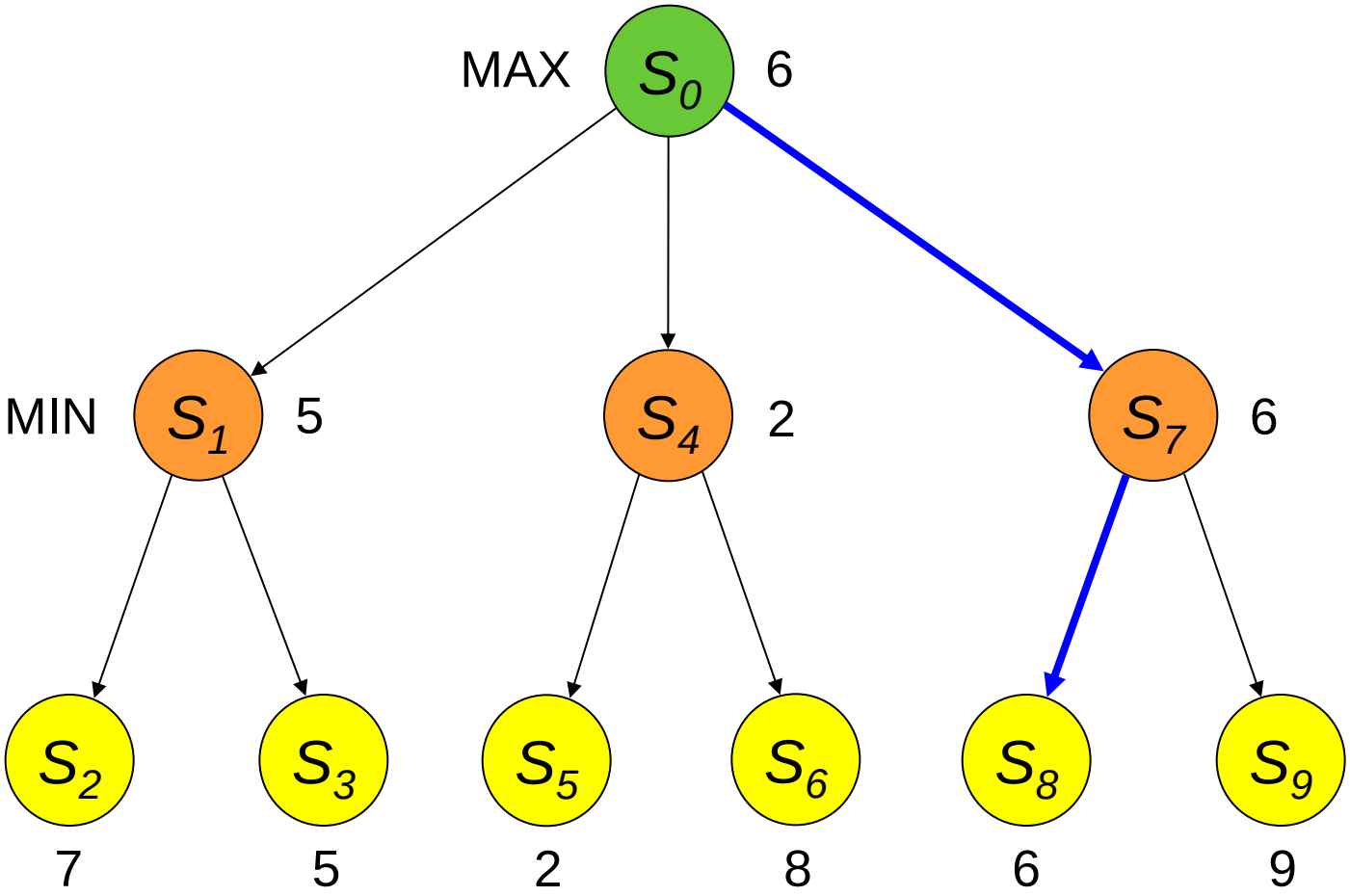
The Minimax Algorithm



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Pruning the Search

- Minimax is an exhaustive search algorithm, so it is exponential in the number of moves, i.e. $O(b^m)$
- This is, to say the least, not desirable
- We can only apply full blown minimax to very small games, or games which are close to a terminal state
- However, you may have noticed in the previous example that we don't have to explore the entire tree in order to find the optimal move…

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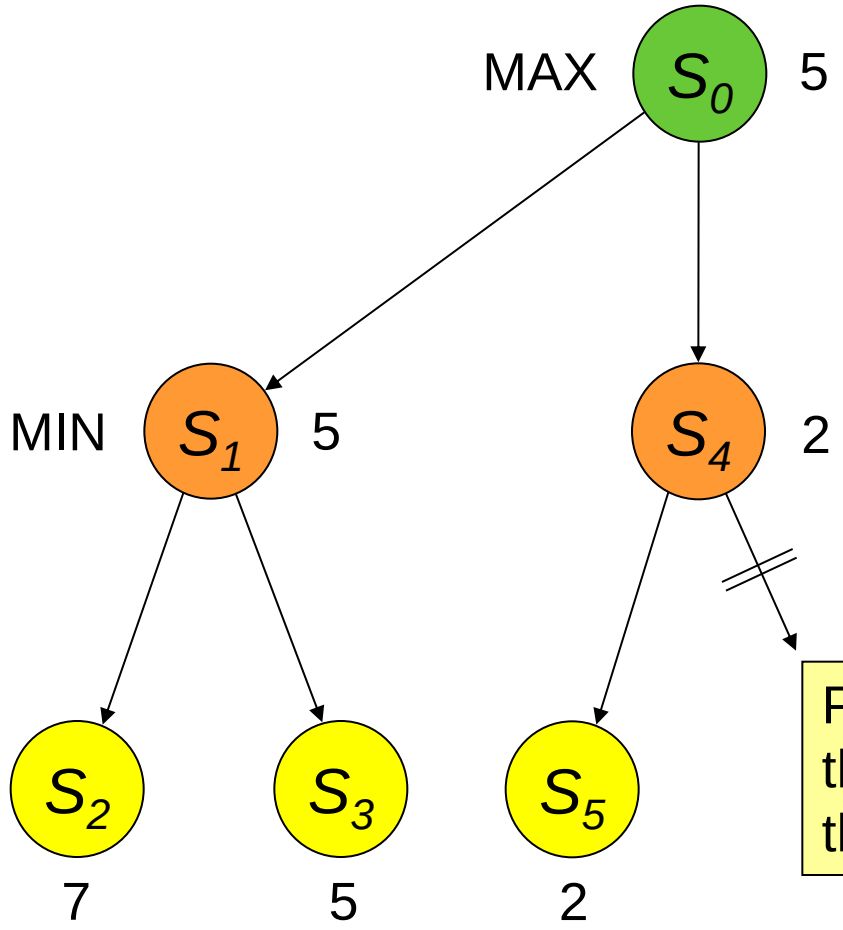
Pruning the Search



Prune further search along this branch and return 2 as the minimax value of S_4

Alpha-Beta Pruning

- Alpha-Beta pruning is a method for ignoring branches of the search tree, while still finding the optimal move
- Allows us to find the optimal move much more quickly
- More on this in next Thursday's lecture…