Static analysis over tree-structured data using graph decompositions

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Contains joint work with Mikołaj Bojańczyk, Wojciech Czerwiński, Claire David, Filip Mazowiecki, Pawel Parys, and Adam Witkowski.

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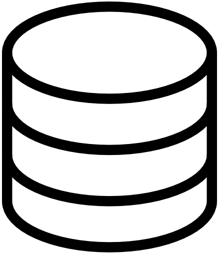
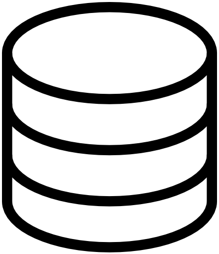
Problems

Data



Data

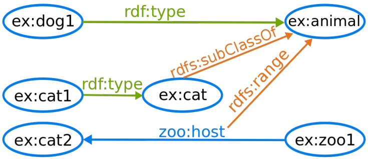


Students Table

Student	ID*
John Smith	084
Jane Bloggs	100
John Smith	182

Activities Table

ID*	Activity*	Cost
084	Swimming	$17
084	Tennis	$36
100	Squash	$40
100	Swimming	$17
182	Tennis	$36




```
<xypair>
    <xaxis axistype="independent">
        <property>Wavelength</property>
        <value>254.0</value>
        <unit>nm</unit>
    </xaxis>
    <yaxis axistype="dependent">
        <property>Absorbance</property>
        <value>0.1234</value>
    </yaxis>
</xypair>
```

Data trees

```
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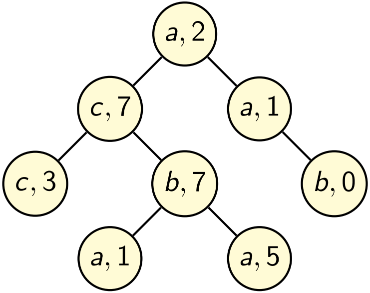

trees finite, unranked, ordered

labels $a, b, c, \ldots$ from a finite alphabet (tags)

data values $0, 1, 2, \ldots$ from an infinite data domain (contents)

Schemas describe allowed shapes of data trees

```
<xs:element name="brightstar">
  <xs:complexType>
    <xs:sequence>
      <xs:element name="name" type="xs:string"/>
      <xs:element name="magnitude" type="xs:decimal"/>
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    </xs:sequence>
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 </xs:element>
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Define several types of trees, each specified (recursively) by

- the label of the root,
- possible sequences of immediate subtree types (regexp);

and choose some of the types as allowed.

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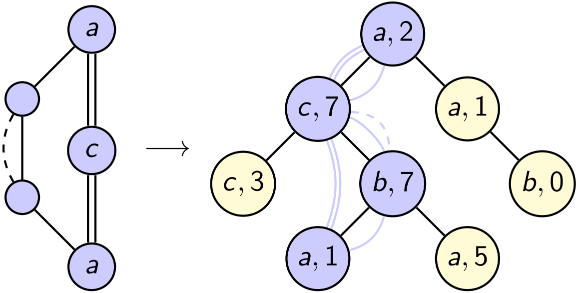
Example: *a*-only path from root to leaf, *b*'s elsewhere

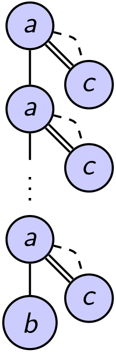
- type τ: root label *a*, immediate subtree types $\sigma^*\tau\sigma^* + \epsilon$;
- type σ: root label *b*, immediate subtree types σ^* ;
- choose: τ .

Conjunctive queries over data trees



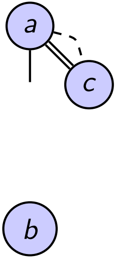
$$\exists x_1 \cdots \exists x_5$$
$$\begin{aligned} child(x_1, x_2) &\wedge child(x_2, x_3) \wedge child(x_3, x_4) \wedge \\ &\wedge desc(x_1, x_5) \wedge desc(x_5, x_4) \wedge \\ &\wedge a(x_1) \wedge a(x_4) \wedge c(x_5) \wedge \\ &\wedge x_2 \sim x_3 \end{aligned}$$

Datalog on data trees



$$p(x) \leftarrow a(x) \land desc(x,y) \land c(y) \land x \sim y \land child(x,z) \land p(z)$$

$$p(x) \leftarrow b(x)$$



extensional predicates *child*, *desc*, $\sim$, *a*, *b*, *c*, ...;

intensional predicates defined recursively using conjunctive queries;

monadic only unary intensional predicates;

linear at most one intensional atom per rule.

Static analysis problems

Satisfiability: Is query P (CQ, UCQ, Datalog, FO, etc.) satisfied in some data tree (conforming to given schema)?

Equivalence: Are queries P, Q equivalent on all data trees?

Containment: Does P imply Q on all data trees?

The staple of data management: query optimization, consistency tests, evaluation modulo constraints, constraint entailment, ...

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P sat	iff	not $P \Leftrightarrow \bot$	iff	not $P \Rightarrow \bot$
$P \wedge \neg Q$, $Q \wedge \neg P$ unsat	iff	$P \Leftrightarrow Q$	iff	$P \Rightarrow Q$, $Q \Rightarrow P$
$P \wedge \neg Q$ unsat	iff	$P \Leftrightarrow P \wedge Q$	iff	$P \Rightarrow Q$

Problems

Old solutions

New solution

More problems with solutions

Some problems without solutions

Containment of CQs over arbitrary structures

[Chandra, Merlin '77]

Def: $Q \in \mathrm{CQ}$ $\rightsquigarrow$ $\mathbb{A}_Q$: universe $\mathrm{Var}Q$, relations given by atoms of Q

Fact: $\mathbb{A} \models Q$ iff exists $h\colon \mathbb{A}_Q \to \mathbb{A}$

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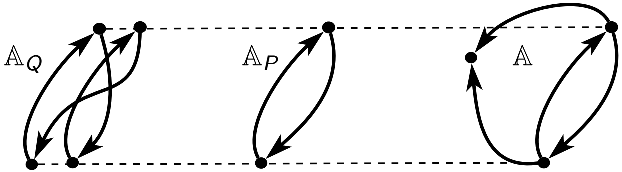

($\Leftarrow$) If $g\colon \mathbb{A}_Q \to \mathbb{A}_P$ and $h\colon \mathbb{A}_P \to \mathbb{A}$, then $h \circ g\colon \mathbb{A}_Q \to \mathbb{A}$.

($\Rightarrow$) $\mathbb{A}_P \models P$ and $P \Rightarrow Q$, so $\mathbb{A}_P \models Q$. Exists $h : \mathbb{A}_Q \to \mathbb{A}_P$.

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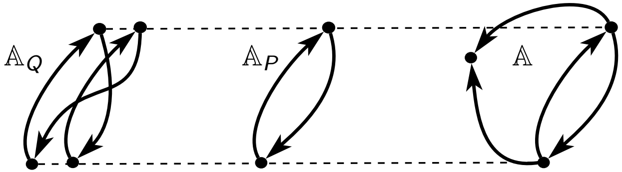

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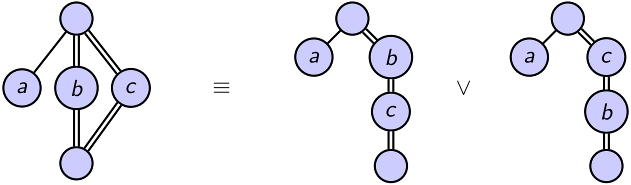
To decide containment, test existence of a homomorphism.

Containment for UCQs over trees without data

[Miklau, Suciu '04]

Each UCQ is equivalent to a union of tree-shaped CQs:



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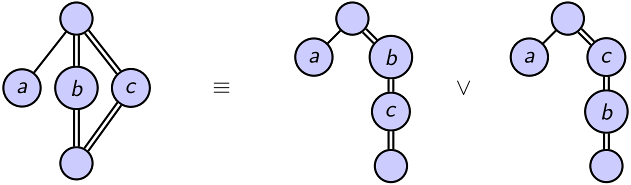

For a tree shaped CQ π build an equivalent tree automaton:

- it computes bottom-up the set of matched subtrees of π;
- knowing which subtrees of π match at the children of node v or strictly below, one can tell which match at v or strictly below.

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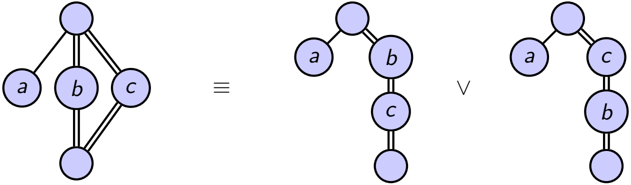

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Tree automata are effectively closed under Boolean combinations.

Test emptiness of the automaton corresponding to $P \wedge \neg Q$.

Containment for UCQs over data trees

[Björklund, Martens, Schwentick '08]

Can restrict to trees with data values $c_1, \ldots, c_{\|P\|}$ and distinct nulls.

- Let T be a tree satisfying P and not Q.
- P touches $\leq \|P\|$ data values in T; replace with $c_1, \ldots, c_{\|P\|}$.
- In each node not touched by P put a unique fresh data value.
- The resulting tree T' still satisfies P and not Q.

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In such trees, $x \sim y$ holds iff either $x = y$ or $x \sim c_i$ and $y \sim c_i$.

By considering all possibilities, replace P, Q with P', Q' using only $x = y$, $x \sim c_i$, $y \sim c_i$.

Check containment over the finite alphabet $\Sigma \times \{\bot, c_1, \ldots, c_n\}$.

Equivalence for Datalog

Equivalence for Datalog is undecidable:

- with descendant [Abiteboul, Bourhis, Muscholl, Wu 2013]
- for non-linear programs [Mazowiecki, Murlak, Witkowski 2014]
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Theorem (Mazowiecki, Murlak, Witkowski 2014)

Equivalence for linear monadic Datalog without desc is decidable.

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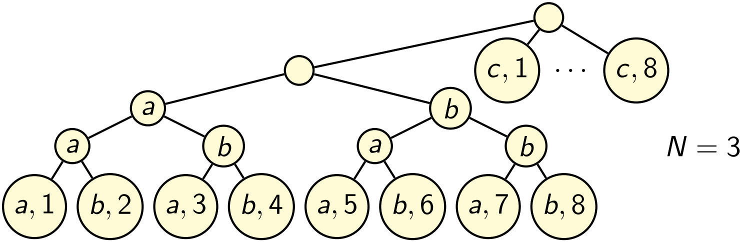
Theorem (Mazowiecki, Murlak, Witkowski 2014)

Equivalence for linear monadic Datalog without desc is decidable.

Can't we restrict reused datavalues like before?

- Let T be a tree satisfying P and not Q.
- Then T satisfies some CQ P_0, an unravelling of P.
- P_0 touches $\leq \|P_0\|$ data values in T, like before,
- but $\|P_0\|$ can be arbitrarily large...

Example



$$
\begin{aligned}
P &\leftarrow DOWN_0(x)\\
DOWN_i(x) &\leftarrow child(x,y) \wedge a(y) \wedge DOWN_{i+1}(y)\\
DOWN_N(x) &\leftarrow UP_N(x) \wedge (N{+}1)\text{-}parent(x,y) \wedge child(y,z) \wedge c(z) \wedge x \sim z\\
UP_i(x) &\leftarrow a(x) \wedge parent(x,y) \wedge child(y,z) \wedge b(z) \wedge DOWN_i(z)\\
UP_i(x) &\leftarrow b(x) \wedge parent(x,y) \wedge UP_{i-1}(y)\\
UP_0(x) &\leftarrow true\\
Q &\leftarrow x \sim y \wedge i\text{-}parent(x,x') \wedge i\text{-}parent(y,y') \wedge a(x') \wedge b(y')
\end{aligned}
$$

Problems

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Some problems without solutions

Clique-width

Instead of processing structures, process their hierarchical decompositions (derivations).

Construct (derive) coloured structures using operations:

i – create a new node of colour i;

$R(i_1, \dots, i_r)$ – add to R all tuples of nodes with colours $(i_1, \dots, i_r)$;

$i \mapsto j$ – change colour i to j;

$\oplus$ – take disjoint union of two structures.

clique-width$(\mathbb{A})$ = least number of colours sufficient to construct $\mathbb{A}$

Examples

Linear orders: clique-width 2

yellow

Examples

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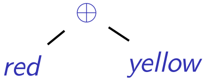

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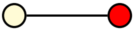

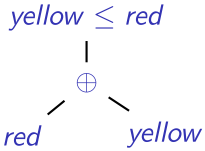

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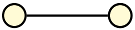

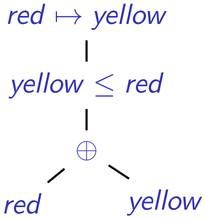

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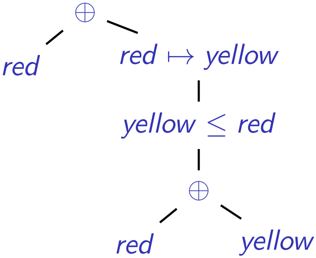

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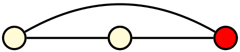

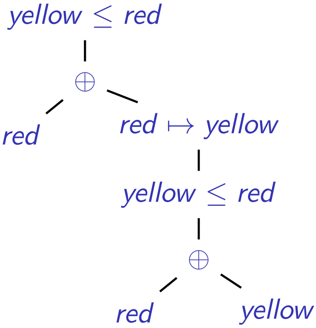

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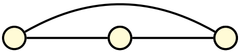

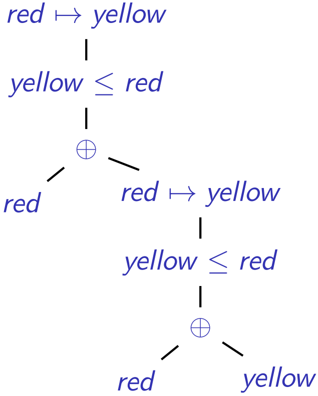

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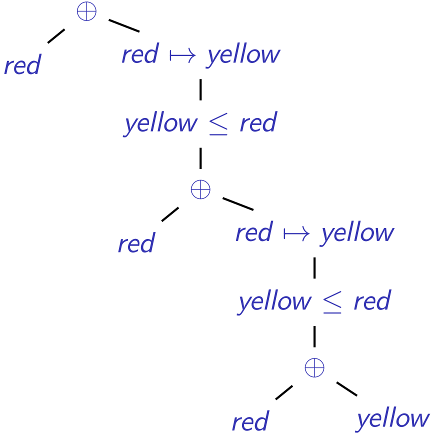

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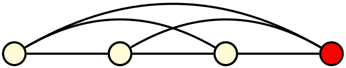

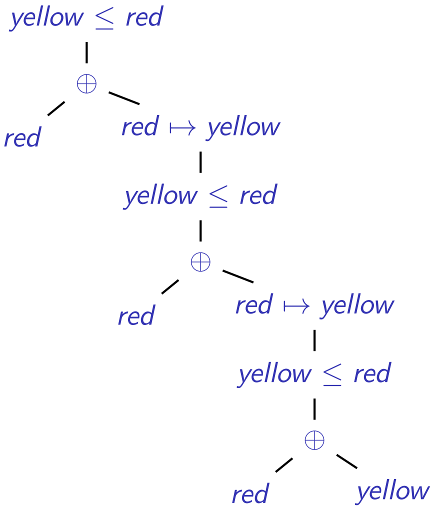

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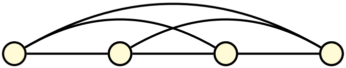

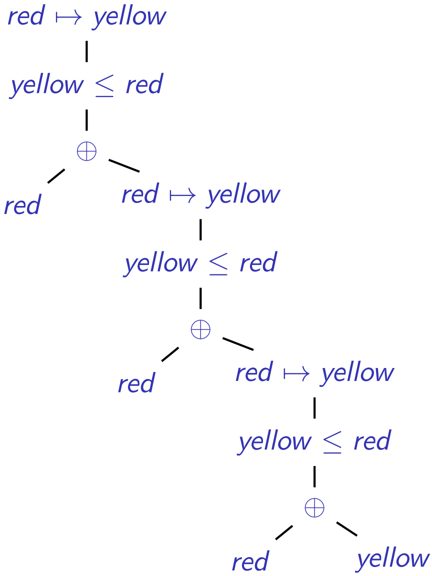

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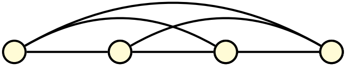
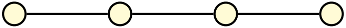
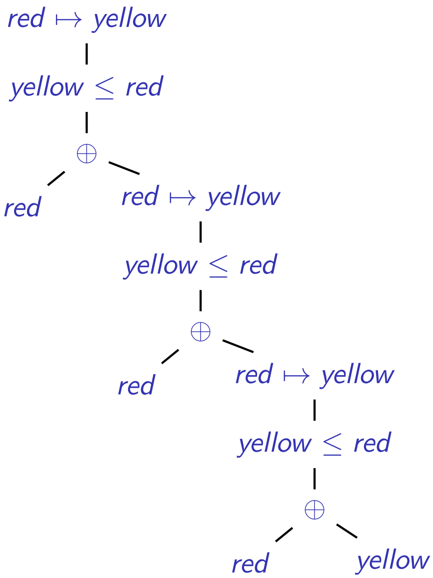


Paths: clique-width 3





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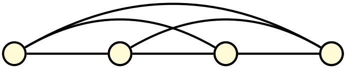

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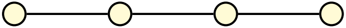
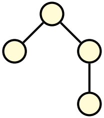
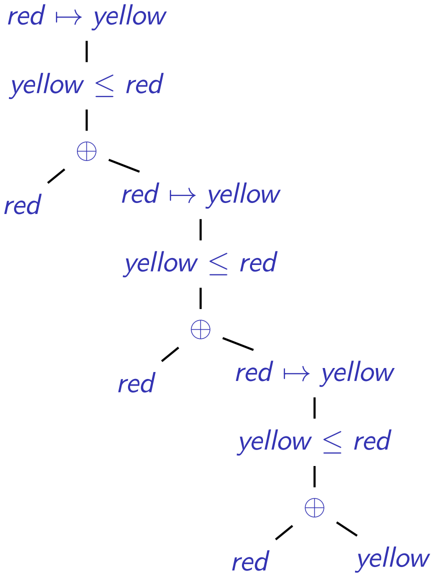


Trees: clique-width 3





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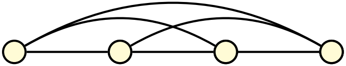

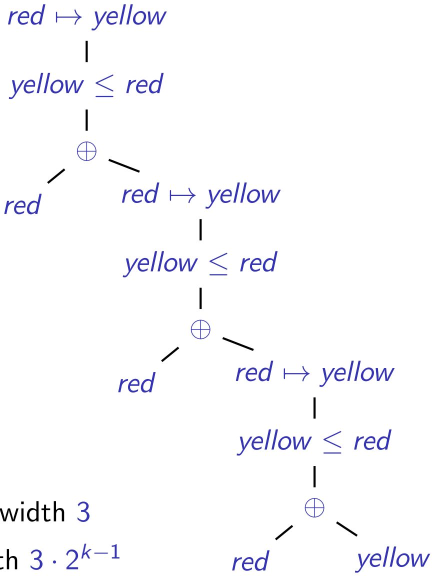

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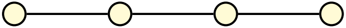

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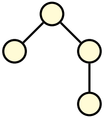

Cographs: clique-width 2

Distance-hereditary graphs: clique-width 3

Graphs of tree-width k: clique-width $3 \cdot 2^{k-1}$

Bounded clique-width means simple

Many NP-complete problems are in P for graphs of bounded clique-width.

Fixed-parameter tractable with clique-width as parameter: time $f(k) \cdot n^c$ on inputs of size n and clique-width at most k, where f is some function, and c is an absolute constant.

Hamiltonicity

Is there a path in graph G that visits each node exactly once?

3-colorability

Can nodes of the graph G be coloured so that each edge connects nodes of different colours?

Courcelle's theorem

Monadic second order logic (MSO)

$$\varphi, \psi \ ::= \ R(x_1, \ldots, x_r) \ \mid \neg\varphi \ \mid \ \varphi \wedge \psi \ \mid \ \varphi \vee \psi \ \mid \ \exists x\, \varphi \ \mid \ \forall x\, \varphi \ \mid \ \mid \exists X\, \varphi \ \mid \ \forall X\, \varphi \ \mid \ X(x)$$

3-colorability

$$\exists X_1\, \exists X_2\, \exists X_3 \quad \forall x\, X_1(x) \vee X_2(x) \vee X_3(x) \wedge \forall x\, \forall y\, E(x,y) \Rightarrow \bigwedge_i \neg(X_i(x) \wedge X_i(y))$$

Theorem (Courcelle)

For every $k \in \mathbb{N}$ and $\varphi \in MSO$ one can construct an automaton recognizing k-derivations yielding models of φ.

Courcelle's theorem applied to parametrized complexity

Theorem (Courcelle)

For every $k \in \mathbb{N}$ and $\varphi \in MSO$ one can construct an automaton recognizing k-derivations yielding models of φ.

Corollary

Each set of structures definable in MSO can be decided in polynomial time over graphs of bounded cliquewidth.

- Compute k-derivation e for the input structure (poly-time);
- construct the automaton $\mathcal{A}$ for k and the defining formula φ;
- run the automaton $\mathcal{A}$ on e.

Courcelle's theorem applied to static analysis

Theorem (Courcelle)

For every $k \in \mathbb{N}$ *and* $\varphi \in MSO$ *one can construct an automaton recognizing* k*-derivations yielding models of* φ*.*

Corollary

For every $k \in \mathbb{N}$*, it is decidable if given* $\varphi \in MSO$ *has a model of clique-width at most* k*.*

- Construct the automaton $\mathcal{A}$ for k and the formula φ;
- test emptiness of the automaton $\mathcal{A}$ (poly-time).

Datalog containment via bounded clique-width

[Bojańczyk, Murlak, Witkowski '15]

Theorem

Let P, Q be monadic, linear Datalog programs without descendant. If $P \wedge \neg Q$ is satisfiable, it is satisfiable in a data tree of clique-width at most $10 \cdot \|P\|^2$.

Corollary

Containment for linear monadic Datalog programs without descendant is decidable.

- Rewrite monadic programs P, Q into $\varphi_P, \varphi_Q \in MSO$.
- Write $\varphi_{datatree} \in MSO$ saying that the structure is a data tree.
- Test satisfiability of $\varphi_P \wedge \neg\varphi_Q \wedge \varphi_{datatree}$.
- For tight complexity, adjust Courcelle's theorem to Datalog.

Problems

Old solutions

New solution

More problems with solutions

Some problems without solutions

Containment for downward Datalog

[Bojańczyk, Murlak, Witkowski '15]

A monadic Datalog program is downward if in all rules for $S(x)$, all mentioned nodes are descendants of x.

Theorem

Let P, Q be downward Datalog programs. If $P \wedge \neg Q$ is satisfiable, it is satisfiable in a data tree of clique-width at most $5 \cdot \|P\|$.

Corollary

Containment for downward Datalog programs is decidable.

Non-mixing constraints

[Czerwiński, David, Murlak, Parys '16]

In database systems, correctness of data is expressed with integrity constraints:

$$\varphi(\bar{x}) \Rightarrow \alpha_{\sim}(\bar{x}) \quad \text{and} \quad \varphi(\bar{x}) \Rightarrow \alpha_{\nsim}(\bar{x})$$

with $\varphi \in \mathit{UCQ}(\mathit{child}, \mathit{desc}, \Sigma)$, $\alpha_{\sim} \in \mathit{UCQ}(\sim)$, $\alpha_{\nsim} \in \mathit{UCQ}(\nsim)$.

Validity: Does each data tree of schema $\mathcal{S}$ satisfy set Δ of non-mixing constraints?

Entailment: Does each data tree of schema $\mathcal{S}$ that sastisfies Δ also satisfies constraint δ?

Theorem

Both problems allow counter-examples of bounded clique-width.

Problems

Old solutions

New solution

More problems with solutions

Some problems without solutions

Open problems

Containment of Datalog programs

- in the presence of a schema;
- with sibling order.

Non-mixing constraints with

- free use of comparisons with constants;
- Skolem functions.