Undefinability of Standard Sequent Calculi for 3-valued Paraconsistent Logics

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Abstract

In the present paper we focus on a family of paraconsistent logics including both the Logic of Paradox [10] (LP) and Paraconsistent Weak Kleene logic, PWK [9, 11], which has been recently studied under different perspectives [7, 6].

We aim at investigating their deductive power by furnishing appropriate sequent calculi. Different types of sequent calculi has been introduced for LP [1], [2], [3], [5]. On the other hand, to the authors' best knowledge, the only attempt to provide a sequent calculus for PWK is [8]. All the existing sequent calculi for these paraconsistent three-valued logics present non-standard features, for instance non standard axioms [4], logical rules introducing more than one connective [4], [3] or logical rules that can be applied only in presence of certain linguistic conditions (this is the case in [8]). The main result of this work consists of proving the impossibility of providing *standard*, as well as *quasi-standard* sequent calculi for a family of logics including both LP and PWK. Standard and quasi-standard calculi are differentiated by the linguistic interpretation of sequents, which is displayed in the object language, in standard case, or in the metalanguage, in the quasi-standard one. At the light of the mentioned negative result, we opted for introducing a three-sided sequent calculus for PWK, which we prove to be cut-free and decidable.

PWK has been extensively studied with the tools of Abstract Algebraic Logic in [6]. We wonder whether the above mentioned negative result might have an algebraic counterpart.

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