# Induction-Induction Part 2
## Specifying quotient inductive-inductive types

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In Homotopy Type Theory [Awodey, Warren 2009; Voevodsky 2010]:

- Homotopical models suggest that equality can be given much more intricate *proof-relevant* structure.
- Equality type $\equiv_A$ provides access to this structure, and is morally part of $A$ (cf. `cubicaltt` [Cohen, Coquand, Huber, Mörtberg 2015]).

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Applications:

- Synthetic homotopy theory:
  - Definition of the circle $\mathbb{S}^1$, with $\pi_1(\mathbb{S}^1) = \mathbb{Z}$,
  - Higher spheres $\mathbb{S}^n$,
  - The Hopf fibration, ...
- Quotidian applications:
  - Cauchy Reals $\mathbb{R}_\mathsf{c}$,
  - the Partiality monad $(-)_\perp$,
  - Type Theory in Type Theory.

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Quotient Inductive-Inductive Types (QIITs): QITs + IITs = QIITs.

All quotidian applications of HITs are QIITs.

# Type theory in type theory as a QIIT

Simplified adaption after Altenkirch and Kaposi [2016]:

$$\textbf{data}\ \mathsf{Con} : \mathsf{Set}$$
$$\textbf{data}\ \mathsf{Ty} : \mathsf{Con} \to \mathsf{Set}$$

$$\varepsilon : \mathsf{Con}$$
$$\mathsf{ext} : (\Gamma : \mathsf{Con}) \to \mathsf{Ty}\,\Gamma \to \mathsf{Con}$$
$$\mathsf{U} : (\Gamma : \mathsf{Con}) \to \mathsf{Ty}\,\Gamma$$
$$\sigma : (\Gamma : \mathsf{Con}) \to (\mathtt{A} : \mathsf{Ty}\,\Gamma) \to \mathsf{Ty}\,(\mathsf{ext}\,\Gamma\,A) \to \mathsf{Ty}\,\Gamma$$
$$\sigma_{eq} : (\Gamma : \mathsf{Con}) \to (\mathtt{A} : \mathsf{Ty}\,\Gamma) \to (\mathtt{B} : \mathsf{Ty}\,(\mathsf{ext}\,\Gamma\,A)) \to (\mathsf{ext}\,(\mathsf{ext}\,\Gamma\,\mathtt{A})\,\mathtt{B} \equiv_{\mathsf{Con}} \mathsf{ext}\,\Gamma\,(\sigma\,\Gamma\,\mathtt{A}\,\mathtt{B}))$$

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$$\sigma : (\Gamma : \mathsf{Con}) \to (A : \mathsf{Ty}\,\Gamma) \to \mathsf{Ty}\,(\mathsf{ext}\,\Gamma\,A) \to \mathsf{Ty}\,\Gamma$$

- "Path constructors" construct equalities, not elements:

$$\sigma_{eq} : (\Gamma : \mathsf{Con}) \to (A : \mathsf{Ty}\,\Gamma) \to (B : \mathsf{Ty}\,(\mathsf{ext}\,\Gamma\,A)) \\ \to (\mathsf{ext}\,(\mathsf{ext}\,\Gamma\,A)\,B \equiv_{\mathsf{Con}} \mathsf{ext}\,\Gamma\,(\sigma\,\Gamma\,A\,B))$$

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Related/alternative work: Kaposi-Kovács [2018].

# High level view

A QIIT is given by a sequence of constructors.

[cat. $\mathcal{A}$ of algebras of previous constructors] $\xRightarrow{\text{specify}}$ [new constructor c on $\mathcal{A}$] $\xRightarrow{\text{extend } \mathcal{A} \text{ with c}}$ [new cat. of algebras $\mathcal{A}'$]

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$$\mathsf{c} : (x : \mathsf{F}(\mathsf{X})) \to \mathsf{G}(\mathsf{X}, \mathsf{x})$$

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Of course, we need restrictions on these functors.

# Well-behaved functors

$$c : (x : F(X)) \to G(X, x)$$

**Argument functor** $F : \mathcal{C} \Rightarrow \mathrm{Set}$ needs to be constrained (strictly positive etc) to prove existence, but can otherwise be arbitrary.

**Target functor** $G : \int^{\mathcal{C}} F \Rightarrow \mathrm{Set}$ definitely cannot be arbitrary.

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category of elements of $F$:
**objects** $(X, x)$, where $X$ in $\mathcal{C}$ and $x : \mathsf{F(X)}$,
**morphisms** $(X, x) \to (X', x')$ consists of
$f : X \to X'$ with $F(f)x \equiv x'$.

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Complication: $\int^{\mathcal{C}} F$ is often not complete, even if $\mathcal{C}$ is, so we need a less vacuous notion of continuity.

# Relative continuity

**Definition** Let $\mathcal{C}$ be a category, $\mathcal{C}_0$ a complete category, and $U : \mathcal{C} \Rightarrow \mathcal{C}_0$.

$$\begin{array}{ccc} \mathcal{C} & \xrightarrow{G} & \mathrm{Set} \\ {\scriptstyle U}\big\downarrow & & \\ \mathcal{C}_0 & \text{(complete)} & \end{array}$$

- A cone in $\mathcal{C}$ is a *$U$-limit cone* if it is mapped to a limit cone by $U$.
- A functor $G : \mathcal{C} \Rightarrow \mathrm{Set}$ is *$U$-relatively continuous* if it maps $U$-limit cones to limit cones in Set.

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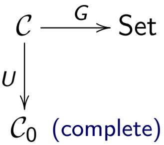


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**Example** Let $U : \int^{\mathcal{C}} F \Rightarrow \mathcal{C}$ be the forgetful functor $U(X, x) = X$. If a functor $G : \int^{\mathcal{C}} F \Rightarrow \mathrm{Set}$ is $U$-relatively continuous, then e.g.

$$G(X \times Y, z) = G(X, z_0) \times G(Y, z_1)$$

where $z_i = F(\pi_i)z$.

# Constructor specifications

**Definition** A constructor specification on a complete category $\mathcal{C}$ is given by

- A functor $F : \mathcal{C} \Rightarrow \mathrm{Set}$ (the *argument functor*).
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The corresponding *category of algebras* $\mathcal{C}.(F, G)$ has

**objects** pairs $(X : \mathcal{C}, f : (x : \mathsf{F(X)}) \to \mathsf{G(X, x)})$

**morphisms** $(X, f) \to (Y, g)$ consisting of $\alpha : X \to Y$ making the obvious "dependent diagram" commute.

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**Example**

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$$F_{\sigma_{eq}}(C, T, ext, \sigma) = (\Sigma\Gamma : C)(\Sigma A : T(\Gamma))(T(ext\,\Gamma\,A))$$
$$G_{\sigma_{eq}}(C, T, ext, \sigma, \Gamma, A, B) = \big((ext\,(ext\,\Gamma\,A)\,B) \equiv_C (ext\,\Gamma\,(\sigma\,\Gamma\,A\,B))\big).$$

# Categories of algebras are complete

**Theorem** Let $(F, G)$ be a constructor specification on a complete category $\mathcal{C}$. Then the category of algebras $C.(F, G)$ is also complete. □

For this, relative continuity of target functors is essential.

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3. Partial progress towards existence of initial algebras (solution set condition missing).

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**Theorem** Target functors for point and path constructors are relatively continuous.

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# Summary

QIITs represented by sequence of constructor specifications.

Constructor specification given by argument and target functors.

Each QIIT representation gives rise to a category of algebras; we are interested in its initial object.

An algebra is initial exactly when it satisfies the usual induction principle.

Same method should work also for higher inductive types, but we want to make sure that all categorical concepts still make sense.

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