

CS103 Machines, Languages and Computation 2015

Python Exercise 1: Collatz Sequences

This is an exercise in turning algorithms into working code using Python. You do **not** have to use recursion in this exercise – the second exercise will use recursion.

Collatz sequences are generated like this: think of any positive integer, which we'll call n . If n is even, divide it by 2, otherwise multiply it by 3 and add 1 to it. Keep on doing this until you get to 1, at which point stop.

For example, starting with $n = 12$, we generate this sequence:

[12, 6, 3, 10, 5, 16, 8, 4, 2, 1]

The length of this sequence is 10 – quite a long sequence, but I'm sure there must be longer ones? My question is this:

For the numbers from $n = 1$ to $n = 100,000$, which starting number gives the longest Collatz sequence and how long is that sequence?

You will solve this in two parts.

Part 1: write the function `collatz(n)`

Write the function `collatz(n)`. This takes as its input the start value of n and calculates the length of the Collatz sequence starting with n using the algorithm shown in Fig 1.

Test your function by comparing its output with these values:

```
>>> collatz(1)
1
>>> collatz(5)
6
>>> collatz(7)
17
>>> collatz(18)
21
```

If you are stuck, here are some bits of Python that you might find useful...

How to define a function:

```
def square(n):
    return n*n
```

Printing out strings and values:

```
print("The value of n is", n)
```

Simple conditionals:

```
if n < 100:  
    print("n is less than 100")
```

Conditionals using else:

```
if a > b:  
    return(a)  
else:  
    return(b)
```

Conditionals using elif:

```
if temp < 20:  
    print("too cold")  
elif temp > 20:  
    print("too hot")  
else:  
    print("just right")
```

Using Blocks of indented code:

```
if k == 3:  
    print("k is 3")  
    k = square(k)  
else:  
    print("k is not 3")  
    k = k + 100  
return(k)
```

Using while loops:

```
while n <= 10:  
    print(n)  
    n = n + 1
```

Testing for exact division:

```
if n % 3 == 0:  
    print("n divides exactly by 3")
```

Part 2: finding the longest sequence for $n = 1$ to $n = 100,000$

You should now have a working function which calculates the length of the Collatz sequence starting with a given input value. Now I'd like to know for what value of n between 1 and 100,000 gives the longest sequence, and how long is that sequence?

Write a function in Python called `biggest_seq(end)`. The starting value of n should be 1 and end should be the last value of n to be examined.

Then call `biggest_seq(100000)` in Python to find what value of n gives the longest sequence. Finally call `collatz(n)` on this value to find the length of this sequence.

To complete the practical, write your name and your findings here:

Name:

The value of n which gives the longest sequence =

The length of this sequence =

Show your values and your code to the demonstrator and they will add your name to the list of those who have completed the practical successfully.

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