



# Private Names in Non-Commutative Logic

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# A Model of Concurrent Processes in the Calculus of Structures

## Concise Semantics: BV

atomic interaction

$$C\{\bar{\alpha} \parallel \alpha\} \longrightarrow C\{I\}$$

seq

$$C\{(P; Q) \parallel (R; S)\} \longrightarrow C\{(P \parallel R); (Q \parallel S)\}$$

switch

$$C\{(P \otimes Q) \parallel R\} \longrightarrow C\{P \otimes (Q \parallel R)\}$$

commutative monoids:  $(P, \parallel, I)$   $(P, \otimes, I)$

monoid:  $(P, ;, I)$

$P$  is provable ( $\vdash P$ ) whenever  $P \longrightarrow I$ .

## Applications: multi-party sessions

**Client:**

$$\overline{p\_begin(Payload)} \parallel \overline{l\_begin(Payload)} ; \\ c\_commit(Time)$$

**Leader:**

$$l\_begin(Payload) ; \\ prepare(Time) ; \\ \overline{p\_commit(Time)} \parallel \overline{c\_commit(Time)}$$

**Participant:**

$$\overline{p\_begin(Payload)} ; \\ \overline{prepare(Time)} ; \\ p\_commit(Time)$$

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$$\left( \begin{array}{l} \overline{p\_begin(Payload)} \parallel \overline{l\_begin(Payload)} \\ c\_commit(Time) \end{array} \right);$$

$\parallel$

$$\left( \begin{array}{l} l\_begin(Payload); \\ prepare(Time); \\ \overline{p\_commit(Time)} \parallel \overline{c\_commit(Time)} \end{array} \right)$$

$\parallel$

$$\left( \begin{array}{l} \overline{p\_begin(Payload)}; \\ prepare(Time); \\ p\_commit(Time) \end{array} \right)$$

$P$  is provable ( $\vdash P$ ) whenever  $P \longrightarrow I$ .

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$$\left( \begin{array}{l} \overline{p\_begin(Payload)} \parallel p\_begin(Payload) \parallel \\ \underline{l\_begin(Payload)} \parallel l\_begin(Payload) \end{array} \right)$$

;

$(\overline{prepare(Time)} \parallel \underline{prepare(Time)})$

;

$$\left( \begin{array}{l} \overline{p\_commit(Time)} \parallel p\_commit(Time) \parallel \\ \underline{c\_commit(Time)} \parallel \underline{c\_commit(Time)} \end{array} \right)$$

$P$  is provable ( $\vdash P$ ) whenever  $P \longrightarrow I$ .

# A Model of Concurrent Processes in the Calculus of Structures

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## Applications: multi-party sessions

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$P$  is provable ( $\vdash P$ ) whenever  $P \rightarrow I$ .

# An Objective Justification for Processes-as-Propositions

## Concise Semantics: BV

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commutative monoids:  $(P, \parallel, I)$   $(P, \otimes, I)$

monoid:  $(P, ;, I)$

Linear implication

$$P \multimap Q = \bar{P} \parallel Q$$

de Morgan dualities

$$\overline{P \otimes Q} = \bar{P} \parallel \bar{Q}$$

$$\overline{P \parallel Q} = \bar{P} \otimes \bar{Q}$$

$$\overline{P; Q} = \bar{P}; \bar{Q}$$

$$\overline{\bar{\alpha}} = \alpha$$

$$\bar{I} = I$$

## Objectivity

Theorem (Guglielmi 2007)

If  $\vdash C\{P \otimes \bar{P}\}$  then  $\vdash C\{I\}$ .

Corollary

Linear implication is a precongruence.

Leader:

$l\_begin(\text{Payload}) ;$

$prepare(\text{Time}) ;$

$(\overline{p\_commit(\text{Time})} \parallel \overline{c\_commit(\text{Time})})$

$\multimap$

Refined leader:

$(l\_begin(\text{Payload}) \parallel prepare(\text{Time})) ;$

$(\overline{p\_commit(\text{Time})} \parallel \overline{c\_commit(\text{Time})})$

# An Objective Justification for Processes-as-Propositions

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## Objectivity

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Corollary

Linear implication is a precongruence.

$$\vdash \left( \begin{array}{l} \overline{l\_begin(Payload)} ; \\ \overline{prepare(Time)} ; \\ (\overline{p\_commit(Time)} \otimes \overline{c\_commit(Time)}) \end{array} \right)$$

$\parallel$

$$\left( \begin{array}{l} (\overline{l\_begin(Payload)} \parallel \overline{prepare(Time)}) ; \\ (\overline{p\_commit(Time)} \parallel \overline{c\_commit(Time)}) \end{array} \right)$$

# Extending with Choice: Multiplicative-Additive System MAV

atomic interaction

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switch

$$C\{(P \otimes Q) \parallel R\} \rightarrow C\{P \otimes (Q \parallel R)\}$$

commutative monoids:  $(P, \parallel, I)$   $(P, \otimes, I)$

monoid:  $(P, ;, I)$

left and right

$$C\{P \oplus Q\} \rightarrow C\{P\} \quad C\{P \oplus Q\} \rightarrow C\{Q\}$$

external

$$C\{(P \& Q) \parallel R\} \rightarrow C\{(P \parallel R) \& (Q \parallel R)\}$$

tidy

$$C\{I \& I\} \rightarrow C\{I\}$$

de Morgan dualities

$$\overline{P \otimes Q} = \overline{P} \parallel \overline{Q} \quad \overline{P \parallel Q} = \overline{P} \otimes \overline{Q}$$

$$\overline{P \oplus Q} = \overline{P} \& \overline{Q} \quad \overline{P \& Q} = \overline{P} \oplus \overline{Q}$$

$$\overline{P; Q} = \overline{P}; \overline{Q} \quad \overline{\bar{\alpha}} = \alpha \quad \overline{I} = I$$

## Theorem (Horne 2015)

If  $\vdash C\{P \otimes \bar{P}\}$  then  $\vdash C\{I\}$ .

E.g. **OAuth Server**:

`initiate(app_ID, scope);`

`login_page(app_ID, scope);`

$$I \oplus \left( I \& \left( \begin{array}{l} \overline{\text{authenticate}(\text{name}, \text{password})}; \\ \overline{\text{authorisation\_code}(\text{code})}; \\ \text{exchange}(\text{app\_ID}, \text{secret}, \text{code}); \\ I \& \overline{\text{access\_token}(\text{token})} \end{array} \right) \right)$$



# Implication in the Spectrum of Preorders over Processes

## interleaving

*autoconcurrency*  
 $tr(a ; a) = tr(a \parallel a)$

*small serialisable systems*

## traces

$tr(P) \subseteq tr(Q)$

## linear time

$tr(a ; (b \oplus c))$   
 $= tr((a ; b) \oplus (a ; c))$

*static environment*

## simulation

$P \leq Q$

## pomset traces

$ideals(P) \subseteq ideals(Q)$

## branching time

$a ; (b \oplus c) \not\rightarrow (a ; b) \oplus (a ; c)$   
 $(a ; b) \oplus (a ; c) \rightarrow a ; (b \oplus c)$

*mobile environment*

## implication

$\vdash P \rightarrow Q$

## causal

*no autoconcurrency*  
 $a \parallel a \not\rightarrow a ; a$   
 $a ; a \rightarrow a \parallel a$

*action refinement*

*scalable systems*

*history preserving*

## Extending for Private Names and Value Passing



$a(x) \parallel \bar{a}b$  terminates without deadlock.

Proof:

$$\exists x. ax \parallel \bar{a}b \longrightarrow ab \parallel \bar{a}b \longrightarrow I$$

## What about $\forall$ for Private Names?

$$\begin{array}{c} \exists x.P \\ \circ \\ \downarrow \\ \forall x.P \end{array}$$

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$$\overline{\forall x.P} = \exists x.\overline{P}$$

$$\overline{\exists x.P} = \forall x.\overline{P}$$

---

no distributivity over parallel composition

$$\nu x.\overline{ax} \parallel \nu y.\overline{ay}$$

$$\downarrow \overline{a(x)}$$

$$\nu y.\overline{ay}$$

$$\downarrow \overline{a(y)}$$

$$1$$

$$\nu x.(\overline{ax} \parallel \overline{ax})$$

$$\downarrow \overline{a(x)}$$

$$\overline{ax}$$

$$\downarrow \overline{ax}$$

$$1$$

Wrongly:

$$\vdash (\forall x.\overline{ax}) \parallel (\forall y.\overline{ay}) \not\rightarrow \forall x.(\overline{ax} \parallel \overline{ax})$$

---

diagonalisation

Wrongly:

$$\vdash \forall x.\forall y.P(x, y) \not\rightarrow \forall x.P(x, x)$$

## What about a Self-dual Nominal Quantifier $\nabla$ ?

$$\exists x.P$$

$$\nabla x.P$$

$$\forall x.P$$

$$\overline{\nabla x.P} \equiv \nabla x.\overline{P}$$

no distributivity over parallel composition

$$\nu x.\overline{ax} \parallel \nu y.\overline{ay}$$

$$\downarrow \overline{a(x)}$$

$$\nu y.\overline{ay}$$

$$\downarrow \overline{a(y)}$$

$$1$$

$$\nu x.(\overline{ax} \parallel \overline{ax})$$

$$\downarrow \overline{a(x)}$$

$$\overline{ax}$$

$$\downarrow \overline{ax}$$

$$1$$

Wrongly:

$$\vdash \nabla x.(\overline{ax} \parallel \overline{ax}) \not\rightarrow (\nabla x.\overline{ax}) \parallel (\nabla y.\overline{ay})$$

diagonalisation

Correctly (Roversi 2011):

$$\nabla x.\nabla y.P(x, y) \not\rightarrow \nabla x.P(x, x)$$

# A Pair of De Morgan Dual Nominals $\exists$ and $\forall$ !



$$\overline{\forall x.P} \equiv \exists x.\overline{P}$$

$$\overline{\exists x.P} \equiv \forall x.\overline{P}$$

no distributivity over parallel composition

$$\nu x.\overline{ax} \parallel \nu y.\overline{ay}$$

$$\downarrow \overline{a(x)}$$

$$\nu y.\overline{ay}$$

$$\downarrow \overline{a(y)}$$

$$1$$

$$\nu x.(\overline{ax} \parallel \overline{ax})$$

$$\downarrow \overline{a(x)}$$

$$\overline{ax}$$

$$\downarrow \overline{ax}$$

$$1$$

Correctly:

$$\forall x.(\overline{ax} \parallel \overline{ax}) \not\equiv (\forall x.\overline{ax}) \parallel (\forall y.\overline{ay})$$

and

$$(\forall x.\overline{ax}) \parallel (\forall y.\overline{ay}) \not\equiv \forall x.(\overline{ax} \parallel \overline{ax})$$

diagonalisation

Correctly:

$$\forall x.\forall y.P(x, y) \not\equiv \forall x.P(x, x)$$

# Notice $\exists$ is Necessary for Implication



$$\overline{\exists x.P} \equiv \forall x.\overline{P}$$

$$\overline{\forall x.P} \equiv \exists x.\overline{P}$$

no distributivity over parallel composition

$$\nu x.\overline{ax} \parallel \nu y.\overline{ay}$$

$$\downarrow \overline{a(x)}$$

$$\nu y.\overline{ay}$$

$$\downarrow \overline{a(y)}$$

$$1$$

$$\nu x.(\overline{ax} \parallel \overline{ax})$$

$$\downarrow \overline{a(x)}$$

$$\overline{ax}$$

$$\downarrow \overline{ax}$$

$$1$$

Correctly:

$$\nu \exists x.(ax \otimes ax) \parallel (\exists x.\overline{ax}) \parallel (\exists y.\overline{ay})$$

and

$$\nu (\exists x.ax \otimes \exists y.ay) \parallel \exists x.(\overline{ax} \parallel \overline{ax})$$

diagonalisation

Correctly:

$$\nu \exists x.\exists y.\overline{P(x,y)} \parallel \exists x.P(x,x)$$

## Semantics of MAV1: Rules determined by Cut Elimination

$C\{\forall x.P \parallel R\} \rightarrow C\{\forall x.(P \parallel R)\}$  only if  $x \# R$  (extrude1)     $C\{\forall x.I\} \rightarrow C\{I\}$  (tidy1)

$C\{\forall x.(P ; S)\} \rightarrow C\{\forall x.P ; \forall x.S\}$  (medial1)     $C\{\exists x.P\} \rightarrow C\{P[Y/x]\}$  (select1)

---

$C\{Ix.P \parallel \exists x.Q\} \rightarrow C\{Ix.(P \parallel Q)\}$  (close)     $C\{Ix.I\} \rightarrow C\{I\}$  (tidy name)

$C\{Ix.P \parallel R\} \rightarrow C\{Ix.(P \parallel R)\}$  only if  $x \# R$  (extrude new)

$C\{\exists x.P\} \rightarrow C\{Ix.P\}$  (fresh)     $C\{Ix.\exists y.P\} \rightarrow C\{\exists y.Ix.P\}$  (new wen)

$C\{Ix.(P ; S)\} \rightarrow C\{Ix.P ; Ix.S\}$  (medial new)

$C\{\exists x.P \circ \exists x.S\} \rightarrow C\{\exists x.(P \circ S)\}$  where  $\circ \in \{\parallel, ;\}$  (medial wen)

$C\{\exists x.P \circ R\} \rightarrow C\{\exists x.(P \circ R)\}$  where  $\circ \in \{\parallel, ;\}$  only if  $x \# R$  (left wen)

$C\{R \circ \exists x.Q\} \rightarrow C\{\exists x.(R \circ Q)\}$  where  $\circ \in \{\parallel, ;\}$  only if  $x \# R$  (right wen)

$C\{\forall x.\forall y.P\} \rightarrow C\{\forall y.\forall x.P\}$  for  $\forall \in \{I, \exists\}$  (all name)

$C\{\forall x.P \& \forall x.S\} \rightarrow C\{\forall x.(P \& S)\}$  for  $\forall \in \{I, \exists\}$  (with name)

$C\{\forall x.P \& R\} \rightarrow C\{\forall x.(P \& R)\}$  only if  $x \# R$  for  $\forall \in \{I, \exists\}$  (left name)

$C\{R \& \forall x.Q\} \rightarrow C\{\forall x.(R \& Q)\}$  only if  $x \# R$  for  $\forall \in \{I, \exists\}$  (right name)

# Equivariance is a Design Decision

equivariance for  $\nu$

$$\nu y. \nu x. P \sim \nu x. \nu y. P$$

equivariance in MAV1

$$\mathbb{I}y. \mathbb{I}x. P \equiv \mathbb{I}x. \mathbb{I}y. P$$

$$\exists y. \exists x. P \equiv \exists x. \exists y. P$$

Linear implication respects transitions, e.g.:

$$a(x).b(y) \parallel \nu y. \nu x. \bar{a}x. \bar{b}y$$

$\downarrow \tau$

$$\nu x. (b(y) \parallel \nu y. \bar{b}y)$$

$$\vdash \mathbb{I}x. (\exists y. by \parallel \mathbb{I}y. \bar{b}y) \multimap \exists x. (ax ; \exists y. by) \parallel \mathbb{I}y. \mathbb{I}x. (\bar{a}x ; \bar{b}y)$$

Proof:

$$\exists x. (\forall y. \bar{b}y \otimes \exists y. by) \parallel \exists x. (ax ; \exists y. by) \parallel \mathbb{I}y. \mathbb{I}x. (\bar{a}x ; \bar{b}y)$$

equivariance and extrude  $\mathbb{I}x$

$$\exists x. (\forall y. \bar{b}y \otimes \exists y. by) \parallel \mathbb{I}x. (\exists x. (ax ; \exists y. by) \parallel \mathbb{I}y. (\bar{a}x ; \bar{b}y))$$

instantiate  $\exists x$

$$\exists x. (\forall y. \bar{b}y \otimes \exists y. by) \parallel \mathbb{I}x. ((ax ; \exists y. by) \parallel \mathbb{I}y. (\bar{a}x ; \bar{b}y))$$

push  $\mathbb{I}y$  in and sequence

$$\exists x. (\forall y. \bar{b}y \otimes \exists y. by) \parallel \mathbb{I}x. ((ax \parallel \mathbb{I}y. \bar{a}x) ; (\exists y. by \parallel \mathbb{I}y. \bar{b}y))$$

extrude  $\mathbb{I}y$  and interact

$$\mathbb{I}x. (\exists y. by \parallel \mathbb{I}y. \bar{b}y) \multimap \mathbb{I}x. (\exists y. by \parallel \mathbb{I}y. \bar{b}y)$$



# Results: Cut Elimination and its Consequences

## Proposition (decidability)

*System MAV1 is analytic.*

## Lemma (context reduction)

*If  $\vdash P\sigma \parallel R$  yields  $\vdash Q\sigma \parallel R$ , for any predicate  $R$  and substitution  $\sigma$ , then  $\vdash C\{P\}$  yields  $\vdash C\{Q\}$ , for any context  $C\{\ \}$ .*

## Theorem (cut elimination)

*If  $\vdash C\{P \otimes \overline{P}\}$  then  $\vdash C\{I\}$ .*

## Corollary

*Linear implication is a precongruence.*

## Corollary (completed traces)

*If  $\pi$ -calculus (or  $\pi$ I-calculus) process  $P$  has completed trace  $tr$  then  $\vdash \llbracket tr \rrbracket_{\pi} \dashv \dashv \llbracket P \rrbracket_{\pi}$ .*

## Corollary (consistency)

*If  $\vdash C\{a\}$  then  $\not\vdash \overline{C\{a\}}$ .*

## Corollary (conservativity)

*MAV1 is a conservative extension of both BV and MALL1 with mix.*

## Corollary (complexity for names)

*MAV1 where terms are constants and variables only is PSPACE-complete.*

## Corollary (complexity)

*MAV1 with functions in terms (e.g. spi-calculus) is NEXPTIME-complete.*

## Corollary (session types)

*Coherent protocols are multi-party compatible.*

# Conclusions on Predicates-as-Processes in MAV1

- ▶ Linear implication is a **branching-time** preorder that fully respects **causality**.
- ▶ First direct **cut elimination** result in the calculus of structures for  $\forall$  and  $\exists$ .
- ▶ First cut elimination result for a **de Morgan dual** pair of **nominal** quantifiers  $\forall$  and  $\exists$ .
- ▶ Can embed the finite  $\pi$ -calculus in MAV1.
- ▶ Embedding of  $\pi$ -calculus uses half the expressive power of MAV1:

$\pi$ -calculus process	connective	dual connective	test
prefix / sequential composition	;	;	traces
choice	$\oplus$	$\&$	branching time
parallel composition	$\parallel$	$\otimes$	separation / causality
input of term	$\exists$	$\forall$	symbolic term
fresh private name	$\nu$	$\ominus$	private input (internal)