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Title: On the number of zeros of multiplicity r

Abstract: Let $F \in \mathbf{F}[X_1, \ldots, X_m] \setminus \{0\}$ be a multivariate polynomial where $\mathbf{F}$ is any field. The multiplicity of a zero of a multivariate polynomial can be defined; a generalization of the multiplicity of a zero of a univariate polynomial.

One can upper bound the number of zeros counted with multiplicity that F can have over S^m for some finite $S \subseteq \mathbf{F}$ using the generalized Schwartz-Zippel bound; it was suggested by Augot et al. [1] and was recently proved by Dvir et al. [2]. The bound is in terms of the total degree of F.

We present upper and lower bounds in terms of the exponents of the leading monomial (with respect to any lexicographic ordering). The upper bound is an improvement to the generalized Schwartz-Zippel bound. Further, we allow more general point ensembles, namely $S_1 \times \cdots \times S_m \subseteq \mathbf{F}^m$ for finite S_i.

The bounds can be hard to calculate but quite some concrete bounds can be found on http://zeros.spag.dk.

We present closed formulas for some cases, among others, the cases where we give sufficient conditions for when the lower bound equals the upper bound.

See [3] or http://zeros.spag.dk for further information.

[1] Augot, D., Stepanov, M.: "Interpolation based decoding of Reed-Muller Codes", slides from talk at Special Semester on Grbner Bases and Related Methods, RICAM, 2006,
http://www.ricam.oeaw.ac.at/specsem/srs/groeb/download/Augot.pdf.

[2] Dvir, Z., Kopparty, S., Saraf, S., Sudan, M.: "Extensions to the Method of Multiplicities, with applications to Kakeya Sets and Mergers", arXiv:0901.2529v2, 2009, 26 pages.

[3] Geil, O., Thomsen, C.: "On the number of zeros of multiplicity r", arXiv:0912.1436, 2009, 21 pages.