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Title: Binomial Edge Ideals

Abstract: We introduce *binomial edge ideals* attached to a simple graph G and study their algebraic properties.

Definition: Let G be a simple graph on the vertex set $[n] = \{1, \ldots, n\}$, that is to say, G has no loops and no multiple edges. Furthermore let K be a field and $S = K[x_1, \ldots, x_n, y_1, \ldots, y_n]$ be the polynomial ring in $2n$ variables. For $i < j$ we set $f_{ij} = x_i y_j - x_j y_i$. We define the *binomial edge* ideal $J_G \subset S$ of G as the ideal generated by the binomials $f_{ij} = x_i y_j - x_j y_i$ such that $i < j$ and $\{i, j\}$ is an edge of G.

We characterize those graphs for which the quadratic generators form a Gröbner basis in a lexicographic order induced by a vertex labeling. Such graphs are chordal and claw-free. We give a reduced squarefree Gröbner basis for general G. It follows that all binomial edge ideals are radical ideals. Their minimal primes can be characterized by particular subsets of the vertices of G.

This is joint work with J. Herzog and T. Hibi.

References

[1] J. Herzog, T. Hibi, F. Hreinsdottir, T. Kahle and J. Rauh *Binomial ideals and conditional independence statements*, Advances in applied mathematics, in press, available online 19. feb. 2010.