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Title: Multinomial expressions summation asymptotic approximations

Abstract: In the study of probabilistic tournaments, questions arise about the asymptotic behavior of the following type:

Consider a (simple and undirected) graph $G = ([r], E)$ (for some positive integer r). Define a quadratic form $f(x_1, \ldots, x_r)$ as the sum of the monomials $x_j x_l$ $(1 \leq j < l \leq r)$, such that $\{j, l\} \in E$, and for every positive integer n form the multinomial coefficient sum

$$S(n) = \sum_{i_1,\ldots,i_r} \binom{n}{i_1, \ldots, i_r} \frac{1}{2}^{-f(i_1,\ldots,i_r)}$$

(sum over non-negative integers i_1, ..., i_r, such that the multinomial coefficients are defined). The answer turns out to be that

$$S(n) \sim C \cdot \alpha(G)^n$$

where $\alpha(G)$ is the independence number of G, and C is the number of independent sets of size $\alpha(G)$ in G.

I shall present somewhat more general, and also slightly more precise results. The methods are elementary and purely combinatoric. (However, the problem seems to be related to Laplace approximations, relevant for e.g. the analytic function $\sum S(n)x^n$.)